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## DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

## ALGEBRA.

291. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

An empty water tank has two inflow pipes  $A$ ,  $B$ , which begin to flow at the same moment. When  $B$ , the smaller pipe, has discharged  $s$  gallons, and the tank is  $1/n$  filled, water from both pipes is turned off. After  $A$ ,  $B$ , have been idle, each as many hours as would suffice it to perform  $1/m$  the work done previously by the other pipe, the flow, which is of a uniform rate, is resumed and continued till the tank is filled;  $B$  during the second working period has discharged  $t$  gallons. (1) What is the capacity of the tank? (2) What would be the capacity if  $B$  were an outflow pipe?

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let  $C$ =capacity,  $x$ =number of hours for  $A$  to fill tank,  $y$ =number of hours for  $B$  to fill tank. Then  $C/x$ =what  $A$  does in one hour,  $C/y$ =what  $B$  does in one hour.  $sy/C$ =time for  $B$  to discharge  $s$  gallons,  $(C-sn)x/(Cn)$ =time for  $A$  to discharge  $C/n-s$  gallons,  $sx/Cm$ =time  $A$  is idle,  $(C-sn)y/(Cmn)$ =time  $B$  was idle,  $(s+t)y/C$ =total time  $B$  works, and  $(C-s-t)x/C$ =total time  $A$  works.

$$\text{But } \frac{(s+t)y}{C} = \frac{(C-s-t)x}{C} - \left( \frac{(C-sn)y}{Cmn} - \frac{sx}{Cm} \right).$$

$$\therefore \left( \frac{s+t}{C} + \frac{C-sn}{Cmn} \right) y = \left( \frac{C-s-t}{C} + \frac{s}{Cm} \right) x \dots (1).$$

$$\text{Also } \frac{sy}{C} = \frac{(C-sn)x}{Cn}, \text{ or } y = \frac{(C-sn)x}{sn} \dots (2).$$

$$(2) \text{ in } (1) \text{ gives } C = mn(sn-s-t) + 2sn.$$

$$\text{II. } \frac{(C+sn)x}{Cn} = \text{time } A \text{ works before turned off.}$$

$$\frac{(C+s+t)x}{C} = \text{total time } A \text{ works, } \frac{(C+sn)y}{Cmn} = \text{time } B \text{ was idle.}$$

$$\therefore \left( \frac{s+t}{C} + \frac{C+sn}{Cmn} \right) y = \left( \frac{C+s+t}{C} + \frac{s}{Cm} \right) x \dots (3); y = \frac{(C+sn)x}{sn} \dots (4).$$

(4) in (3) gives  $C = mn(sn - s - t) - 2sn$ .

292. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Find the sum of the series  $1^2 + 5^2 + 14^2 + 30^2 + \dots + [\frac{1}{6}n(n+1)(2n+1)]^2$ .

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

The differences and the terms of this special series may be arranged as follows for the first seven terms:

$u_1 = 1^2$	$5^2$	$14^2$	$30^2$	$55^2$	$91^2$	$140^2$
$u_1 = 1$	25	196	900	3025	8281	19600
$\Delta^1 u_1 = .$	24	171	704	2125	5256	11319
$\Delta^2 u_1 = .$	. 147	533	1421	3131	6063	. .
$\Delta^3 u_1 = .$	. . 386	888	1710	2932	. . .	. .
$\Delta^4 u_1 = .$	. . . 502	822	1222	. . .	. . .	. .
$\Delta^5 u_1 = .$	. . . . 320	400	. . .	. . .	. . .	. .
$\Delta^6 u_1 = .$	. . . . . 80	. . .	. . .	. . .	. . .	. .

Compute the series for ten terms, or more, and it will be found that  $\Delta^6 u_1$  are all 80, or constant, therefore all the higher differences vanish. To sum the series we have the value of the leading term and the six leading differences. I have given a general formula for  $S_n$ , on page 163, of THE AMERICAN MATHEMATICAL MONTHLY for August-September, 1906, see equation (E). We have:

$$S_n = nu_1 + \frac{n(n-1)}{2} \Delta^1 u_1 + \frac{n(n-1)(n-2)}{3!} \Delta^2 u_1 + \dots$$

$$+ \frac{n(n-1) \dots (n-6)}{7!} \Delta^6 u_1 \dots (1).$$

From the problem and the above table we have:  $u_1 = 1$ ,  $\Delta^1 = 24$ ,  $\Delta^2 = 147$ ,  $\Delta^3 = 386$ ,  $\Delta^4 = 502$ ,  $\Delta^5 = 320$ , and  $\Delta^6 = 80$ . Substitute numerical values in (1), expand the terms, consolidate like terms, reduce, and we have:

$$S_n = \frac{20n^7 + 140n^6 + 371n^5 + 455n^4 + 245n^3 + 35n^2 - 6n}{1260} \dots (2),$$

$$= \frac{1}{1260} [n(n+1)(n+2)(2n+1)(2n+3)(5n^2+10n-1)].$$